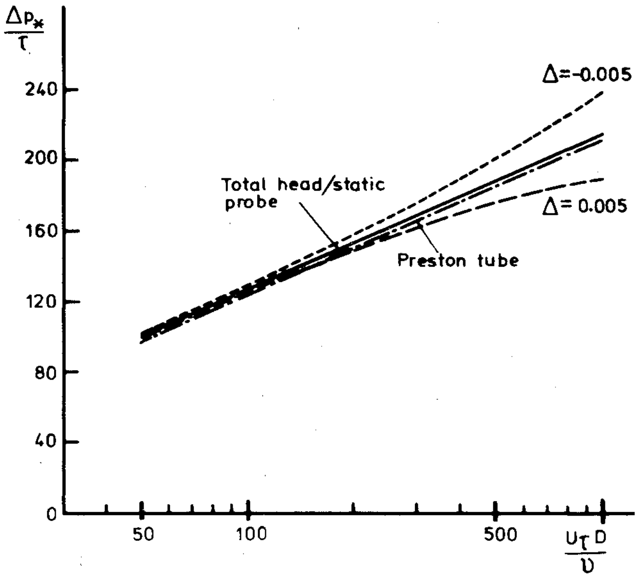

Fig. 2 Calibration curve for total head/static probe relative to Preston tube calibration. The pressure gradient effects are indicated as the additive term $U_\tau D/\nu \cdot L/D \cdot \Delta$, $L/D = 5$. $\Delta = \nu/U_\tau^3 \cdot 1/\rho \cdot dp/dx$.

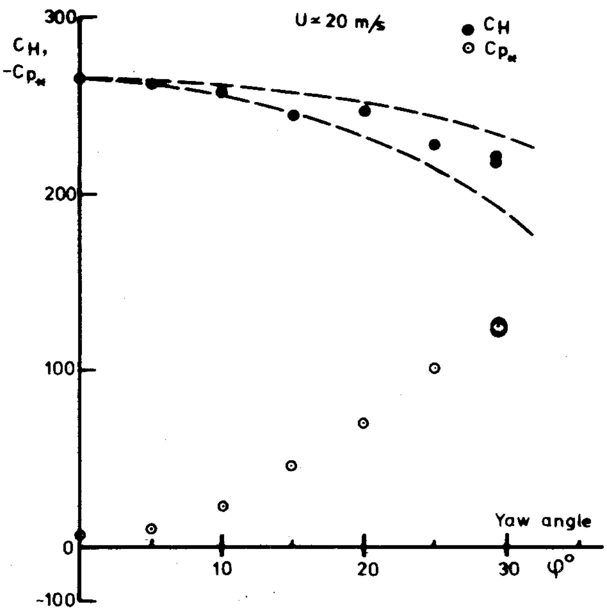

Fig. 3 C_H and C_{p^*} as functions of yaw angle φ. – – Outer limits of data in Ref. 8. It should be noted that the level of C_H corresponds to the Preston tube reading.

This expression is based on $C_{p^*} = -9.2$, a result obtained in zero pressure gradient flow. The addition of the pressure gradient term in Eq. (6) approximately accounts for moderate pressure gradient effects, under the assumption that the pressure gradient will not affect C_{p^*} to first order.

The test range of this probe was $U_\tau D/\nu = 70$ to 200, but the analysis indicates that the calibration is valid at least in the range 50 to 1000. Figure 2 illustrates this curve compared with the corresponding Preston tube curve. The effect of pressure gradient is also indicated.

One might fear that the location of the static pressure tap on top of the cylinder would cause the probe to be sensitive to yaw. A Preston tube in yaw tends to give lower output only due to change in the total pressure p_{tot}, while p_s may be regarded as constant.

However, for the new probe it turned out that the static pressure p_* changed rather slowly at small yaw angles and the response is in the opposite sense, as seen in Fig. 3. $C_H = p_{tot} - p_s / (\rho/2)U_\tau^2$. In the figure the yaw variation of C_H according to Rajaratnam and Muralidhar[8] has been indicated.

Conclusions

A new method for measuring skin friction by means of a total head/static probe in a turbulent boundary layer has been described and a calibration has been given and discussed.

1) The probe measures the total head pressure at the tip and a static pressure at a location downstream of it (in the center plane). By accounting for the position error C_{p^*} both true static pressure and skin friction are determined.

2) The particular probe tested had the configuration $D = 3$ mm and $L/D = 5$, which is to be regarded a reasonable choice for most applications. Variation of the ratio L/D showed that at least down to $L/D = 2$ there is no change in static pressure.

3) The sensitivity to misalignment is about twice that of a Preston tube and the total head/static probe responds in the opposite sense. It should be noted, though, that this statement does not consider the cross flow effects in three-dimensional boundary layers. If the variation in cross flow is small over the probe diameter one may nevertheless expect deviations comparable to the misalignment errors discussed previously.

Acknowledgment

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Boundary Conditions with Heat/Mass Transfer and Velocity Slip

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MOST physical problems of the boundary-layer type which are of general interest to the fluid mechanist

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describe phenomena that make determination of the appropriate boundary conditions relatively straightforward. However, the introduction of simultaneous heat and mass transfer with chemical reactions, phase changes, and velocity slip significantly complicates the situation. The conventional technique for obtaining boundary conditions involves placing a control volume around the boundary plane with subsequent application of appropriate global conservation principles. Quite often this procedure appears disconnected to the system of equations that are being solved and has led to incorrect results because of the absence of a uniformly consistent approach to the problem.

In lieu of applying overall conservation relations to the control volume that surrounds the boundary plane, it is possible to formally integrate the exact governing differential equations in the direction normal to the streamwise coordinate from a location $\epsilon/2$ below the plane to $\epsilon/2$ above it and let ϵ go to zero, assuming that the fluid equations hold throughout the region. Utilizing this procedure, the general multidimensional, nonsteady differential conservation equations result in the following "jump" (δ) conditions across the boundary plane of interest, in a manner identical to that applied to shock waves in Ref. 1

Continuity

$$\delta(\rho v)=0 \tag{1}$$

Streamwise Momentum

$$\delta(\rho u v)=\delta\tau \tag{2}$$

Normal Momentum

$$\delta(\rho v^2)=-\delta p \tag{3}$$

Energy

$$\delta(\rho v E)-\delta(\tau u)+\delta(p v)+\delta(q_y)=0 \tag{4a}$$

or

$$\delta(\rho v H)-\delta(\tau u)+\delta(q_y)=0 \tag{4b}$$

where ρ is the local fluid density, and u, v are the streamwise and normal components of velocity, respectively. The elements of the stress tensor are represented by the static pressure p and the relevant component of shear stress τ. Transverse heat flux is contained in the term q_y whereas the potential energies of reaction or phase change are embedded in a generalized total energy E or enthalpy H.

When applying these conditions to a boundary of interest, it is apparent that any terms that are known to be continuous across it simply drop out of the equations. The resulting expressions provide the desired relationship between the behavior of the unknown terms and those that are being imposed on the solution of the particular problem.

The simplest of these expressions is provided by Eq. (1), which enforces the conservation of mass flux through the boundary plane while allowing both ρ and v to change across it. The streamwise momentum relationship presented in Eq. (2) dictates that the change in shear stress across the boundary is related to the velocity slip, since, as has been noted, ρv is conserved. In the case of flow along a solid boundary in which the zero slip condition is satisfied, the shear stress on the wall therefore is equal to the shear stress in the fluid at the wall, with or without mass transfer.

Conservation of normal momentum, as expressed in Eq. (3), introduces the possibility of a change in pressure across the boundary in the event that the normal velocity changes, for example, in the presence of mass transfer with a change of phase. Of prime importance, however, is the application of conservation of energy in the form of Eq. (4), which illustrates the fact that, without velocity slip or mass transfer (or with isoenthalpic mass transfer), the heat transfer in the fluid at the wall is equal to the wall heat transfer, which allows the respective temperature gradients to be related through Fourier's law. With the introduction of velocity slip, Eq. (4) expresses the relationship

$$q_{y_w}=q_{y_f}-\tau u \tag{5}$$

where a coordinate system at rest with respect to the wall has been utilized. In Eq. (5), τu represents the shear in the fluid at the wall and slip velocity, respectively, whereas the subscripts w and f refer to wall and flowfield quantities. Introduction of nonisoenthalpic mass transfer additionally complicates the energy balance such that Eq. (5) must be written as

$$q_{y_w}=q_{y_f}-\tau u+\delta(mH) \tag{6}$$

where $m\equiv\rho v$, and either or both of the mass transfer terms represent the sum of multiple phase transports, each with various heats of reaction or phase change.

The application of Eq. (6) to an external boundary plane (for example, at the edge of a condensing liquid wall layer being driven by a gas stream) is a situation in which some confusion might arise. Note that, in the true integral solution of the entire liquid film, the shear work term τu must be included explicitly in the energy balance; however, according to Eq. (4), the effect does not enter the external boundary condition evaluation, since both τ and u usually are continuous at the interface of the liquid and gas layers.

It is clear that other equations such as species conservation or extended versions of the basic equations presented here, with additional terms appropriate to specific situations, may be treated in an identical manner. In general, regardless of which equations are being evaluated with respect to boundary condition formulation, the virtue of the technique utilized here becomes apparent only in more complex situations. Although there is no current controversy regarding this problem, it is a fact that, in the past, the absence of a uniform approach to the specification of boundary conditions has resulted in some discrepancies[2-4] with regard to the appropriate form for the heat-transfer boundary condition with velocity slip. To the author's knowledge, the mechanism presented here has not been generally employed for this class of problems, and it is one that, especially for scientists who are being introduced to some of these peculiar and interesting phenomena, provides a significant degree of unity and coherence to the equation/boundary condition formulation.

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Bounds on the Dynamic Characteristics of Rotating Beams

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Introduction

THE dynamic characteristics of rotating beams find application in rotating machinery, helicopters, windmills,

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